Rifling Twist Rate Concerns

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We have been exploiting Coning Theory in the development of our new turned copper Ultra-Low-Drag rifle bullet designs. We ran into an accuracy problem in our test-firing which we did not properly anticipate. Having addressed and ameliorated each of the persistent accuracy issues experienced with conventional and monolithic rifle bullets; in-bore yawing of the engraved bullet, lateral throw-off caused by static or dynamic imbalance of jacketed bullets, gas leakage past monolithic bullets causing varying velocity losses, and a few others, we expected to see improved target accuracy in our latest test results. However, that has not yet occurred.

The Yaw-Drag Problem

David Tubb measured almost **30-percent reduced aerodynamic drag** over 1,000 yards compared to the G7 reference projectile shape, well below the drag predictions of Bob McCoy's McDRAG estimations (well outside his 5-percent estimation limits). We correctly attributed much of this air-drag reduction to reduced yaw-drag in those tests. David used a Schneider 338-caliber barrel with a 7.5-inch twist (**$n = 22.7$ calibers per turn**) which produced an initial gyroscopic stability (**Sg**) of **2.75** with our earlier copper ULD bullets. We correctly intuited that by using a rifling twist-rate of **$n \approx 20$ calibers per turn**, any monolithic bullet of up to 6 calibers in length could fly with its lowest possible total air drag.

The empirical data clearly shows that for dynamically stable rifle bullets we can reduce their air drag by selecting faster twist-rate rifled barrels (smaller values of **n**). For dynamically stable rifle bullets, the slow-mode damping factor **λ_2** (in **inverse seconds**) controls the time required for any initial coning angle-of-attack **α_0** to damp to insignificance. The extra retardation **ΔV** caused by yaw-drag while any initial yaw angle is damping out is ***inversely proportional*** to the size of **λ_2** in seconds of flight time.

The only classical aeroballistics auxiliary parameter (**P**, **M**, **H**, **T**, etc.) which is directly affected by our selection of rifling twist-rate **n** (in calibers per turn) is

$$P = (I_X/I_Y)*[2\pi*V/(n*d)],$$

and **P** is clearly inversely proportional to **n**. Then its sensitivity to **n**, **$\partial P/\partial n$**, must be inversely proportional to the square of **n**.

From linear aeroballistics theory, the sensitivity of **λ_2** to **P**, **$\partial\lambda_2/\partial P$**, shows approximately direct proportionality to the selected twist-rate **n**.

So, the sensitivity of **λ_2** to **n**,

$$\partial\lambda_2/\partial n = (\partial\lambda_2/\partial P)*(\partial P/\partial n),$$

is ***inversely proportional to*** **n**.

Selecting a faster twist-rate barrel (smaller value of **n**), produces a larger slow-mode damping factor **λ_2** and thus a smaller velocity retardation **ΔV**. This extra retardation occurs very early in flight and cannot be recovered. The extra retardation **ΔV** for each shot is proportional to its random initial yaw **α_0** times the twist-rate **n**, or **α_0*n**.

We hypothesize that randomly varying initial aeroballistic yaw, or yaw-rate, is causing the large variation in air-drag measured in David's tests. We attribute this unexpectedly large drag variation to random yaw-destabilization occurring while the rifle bullets are transiting the muzzle-blast region before the commencement of ballistic flight. We have added a convex radiused base onto the boat-tails of our copper bullets which does somewhat mitigate these initial ballistic yaw disturbances.

All this being said, we should point out that the ***shortest*** measured time-of-flight over David's 1,000 yard test range indicates the individual shot which suffered the ***least*** yaw disturbance while transiting the muzzle-blast zone before commencing ballistic flight, so its (highest) calculated **BC** value is most representative of that bullet design's nose-forward air-drag coefficient **CD_0**. The hypothesized, random, non-zero yaw-drag can only slow the bullets and increase their times-of-flight. Beware of using statistical mean values when the random variable has a one-sided distribution function.

The Accuracy Problem

This initial yaw disturbance hypothesis is strongly reinforced by the disappointing accuracy results with these copper ULD bullets test-fired both by David at his 1,000-yard outdoor test range and in our wind-free 100-yard indoor test range. We each are seeing about **0.8 MOA** 5-shot groups with these fast-twist rifle barrels when everything else is done correctly. David's 338-caliber test barrel is rifled at 7.5 inches (**22.7 calibers**) per turn, while ours is rifled at 7.0 inches (**21.2 calibers**) per turn.

The same analysis based on Coning Theory which allowed earlier formulation of the angular trajectory deflection termed "aerodynamic jump" caused by a horizontal crosswind at the firing point holds for a bullet entering a wind-free atmosphere with a non-zero initial aeroballistic yaw attitude. [In fact, this is exactly how Bob McCoy handled the simulation of crosswinds in his own 6-degree-of-freedom simulator.] The resulting angular deflection drives the bullet away from its intended trajectory in a radial direction 90-degrees advanced in the sense of the rifling twist from the roll orientation of the initial yaw angle itself. As this angular deflection is given in milliradians or minutes of angle (MOA), the miss distance produced on the target is strictly proportional to firing distance (minus about 5 or 10 yards where the jump effectively occurs). A random magnitude initial yaw disturbance which is also randomly oriented will simply increase "extreme spread" shot-group sizes as measured on the target.

This aerodynamic jump is caused by an impulsive aerodynamic lift-force moving the CG of the rifle bullet away from its intended trajectory during the first half of its first coning cycle in early ballistic flight. This transient lift-force is integrated over the time duration of the first half-period of the bullet's coning motion to produce a cross-track ***impulse*** (force summed over a short time interval) which shifts the direction of that bullet's linear momentum vector (without changing its magnitude). This rotating lift-force directionally cancels during all subsequent coning motion.

The ***size*** of this aerodynamic lift-force is ***directly proportional*** to the size of the initial yaw angle $\mathbf{a_0}$ causing it. The amount of ***time*** over which this cross-track impulse accumulates is ***inversely proportional*** to the initial ***coning rate*** $\mathbf{f_2}$. The initial coning rate $\mathbf{f_2}$ is determined from the Tri-Cyclic Theory as

$$\mathbf{f_2 = (Ix/Iy)*[V_0/(n*d)]/(R + 1)}$$

where **Ix, Iy =** Second moments of inertia of the bullet's mass distribution about crossed principal axes

d = Caliber of the bullet in feet

$\mathbf{R = f_1/f_2 =}$ Gyroscopic stability ratio.

The gyroscopic stability ratio **R** conveys **1:1** the same information as the classic gyroscopic stability **Sg**, but in a more directly usable form:

$$\mathbf{Sg = (R + 1)^2/(4*R)}$$

Both **Sg** and **R + 1** vary ***inversely with the square*** of the rifle barrel's twist-rate **n** in calibers per turn.

Examining the above expression for $\mathbf{f_2}$, the ***initial coning rate*** $\mathbf{f_2}$ in hertz varies ***directly with the rifling twist-rate*** **n**; i.e., with $\mathbf{n^{-1}/n^{-2}}$, which in turn causes both the cross-track impulse integration ***time*** and indeed the resulting size of that cross-track impulse to vary ***inversely*** with the value of **n**. ***Quicker twist-rates cause proportionally larger aerodynamic jumps*****.**

In examining Bob McCoy's own formulation for aerodynamic jump which is derived by calculus from the Equations of Motion, we find a factor of $\mathbf{2\pi/n}$, as its only dependence upon barrel twist-rate **n**. Both derivations from the Coning Theory and from the Equations of Motion show this same ***inverse dependence*** of the size of the aerodynamic jump upon barrel twist-rate **n**. Thus, in both independent formulations of aerodynamic jump, making **n** smaller (for a faster twist) ***increases*** the sizes of any aerodynamic jump trajectory deflection angles in inverse proportionality.

So, in a practical sense, we could simply say that, all else being equal, "Accuracy is ***directly proportional*** to the twist-rate **n** of the rifle barrel." Competitors in rifle accuracy sports have long sought to use the slowest possible twist-rates (largest number **n**, of perhaps **40 to 60 or more calibers per turn**) in their match rifle barrels. Now we see another rationale supporting that acquired wisdom.

For ***best accuracy*** in the presence of some initial yaw disturbance, we want the slowest possible rifling twist-rate, but for ***lowest air-drag*** with the same initial yaw disturbance we want the much faster **20 calibers per turn** twist-rate, especially in shooting long monolithic ultra-low-drag (ULD) rifle bullets. We cannot have it both ways.

In extreme long-range (ELR) riflery, we need the lowest possible air-drag even at some expense in gilt-edge accuracy, so those ELR riflemen might stick with my recommended **20 calibers per turn** twist-rates with monolithic bullets. On the other hand, 100-yard benchrest competitors will stick with their **60, or more, calibers per turn** 6 mm PPC barrels. I now recommend rifling twist-rates of **24 calibers per turn** for general use with any monolithic rifle bullets.

Takeaways

The problems caused by ***yaw destabilization*** of fired rifle bullets occurring while they are transiting the muzzle-blast zone are much more serious than had been expected. By reducing the initial coning rate from a typical **60 to 75 hertz** for jacketed match bullets to the range of **25 to 45 hertz** for our copper ULD bullets fired from faster twist-rate barrels, we have inadvertently amplified the accuracy problem by up to a **factor of 3**, both in its highly variable extra air-drag aspect and in its angular-jump accuracy destroying effects. We are currently convex-radiusing the boat-tail bases of our copper ULD bullets at **0.74-calibers**, which does help in controlling their yaw destabilization within the muzzle-blast region and also plan to try slightly beveling the rear corners of those boat-tails.

More research is needed into rifle building techniques which facilitate launching monolithic bullets at high speeds from very fast-twist barrels with little or no initial ballistic yaw or yaw-rate. Barrel porting and the use of integral suppressors come to mind, as does trying other non-tubular styles of muzzle brakes. We have acquired a Barrett M98 338-caliber muzzle brake to compare with our very effective tubular MB design. The Barrett design features two large horizontal exit ports per side which should guarantee a very high gas-evacuation-rate. Perhaps artillery designers have already gotten a handle on this yaw-destabilization problem with the high evacuation-rate brakes formerly used before the development of discarding sabots and fin-stabilized projectiles.

Summary of Rifling Twist-Rate Dependence

The aeroballistic parameters which vary in magnitude ***directly with the first power of*** **n**, the rifling twist-rate given in calibers per turn, are:

- Initial coning rate $\boldsymbol{\omega_2(0)}$ in radians per second.

Those parameters which vary ***inversely with the first power of*** **n** are:

- Initial spin-rate $\boldsymbol{\omega(0)}$ of the bullet in radians per second
- Crosswind aerodynamic jump deflection angle $\mathbf{A_J}$ in MOA

- Initial-yaw-caused aerodynamic jump deflection angle **A_J**
- Yaw-of-repose horizontal angle **$\beta_R(t)$** in radians
- Spin-drift **SD(t)** horizontal distance in feet
- Period **T_2** of the first coning cycle **$2\pi/\omega_2(0)$** in seconds.
- Initial slow-mode damping factor **λ_2** for dynamically stable bullets.

Those parameters which vary ***inversely with the square of*** n are:

- Initial gyroscopic stability factor **Sg**
- Initial gyroscopic stability ratio **R**, as (**R + 1**)
- Initial coning CG-to-cone-apex distance **D** in feet
- Initial coning radius **$r = D*\mathrm{Sin}(\alpha)$** of CG motion in feet.

Each aeroballistic coefficient is assumed to vary only with the Mach-speed of the bullet. The mass of the bullet and its spatial distribution parameters are generally assumed to be constant. The muzzle velocity **V_0** of the bullet is assumed to be independent of the rifling twist-rate **n** selected.